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AN ALGORITHM FOR COMPUTING NON-ISOMORPHIC SEMIGROUPS OF FINITE ORDER

by

James Stephen Cullen



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THESIS

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SEMIGROUPS OF FINITE ORDER

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June 1969

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An Algorithm for Computing Non-Isomorphic Semigroups

of Finite Order

by

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Submitted in partial fulfillment of the requirements for the degree of

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from the

Namel Destaraduate School

ABSTRACT

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In this paper an algorithm for computing semigroups of finite order is discussed. A computation procedure is developed to generate, for any specified finite order, all semigroups which are distinct up to isomorphism. Additional restrictions are also placed in the generating procedure to produce all groups of the given finite order. The algorithm was placed on the computer and the numerical results for orders one through four obtained.

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I. INTRODUCTION

In this paper we investigate the problem of computing all possible distinct algebraic systems of a certain type, namely the semigroup, with restrictions on the order only. A semigroup is an algebraic system which is closed and associative, and as such is the simplest algebraic system of significance in mathematics. More complex systems are determined by postulating additional properties, for example commutativity, many of which can easily be placed in a computation procedure. To illustrate this point a procedure was constructed to produce all groups in addition to all semigroups of the specified finite orders. In this paper we consider orders up to and including four only.

II. DEFINITIONS AND PRELIMINARY RESULTS

There are two basic alternatives for defining equivalence of semigroups. One approach is to identify two semigroups if the first is either isomorphic or anti-isomorphic to the second. The resulting collection of distinct semigroups is then described as a collection of nonequivalent semigroups. The alternate approach is to define one semigroup distinct from another if the first is not isomorphic to the second. The resulting collection is then described as a collection of semigroups which are distinct up to isomorphism. We use the latter approach in this paper and will explain the reasons for this choice later.

We begin by recalling the definitions of a binary operation on a set, of some properties a binary operation may possess, and of a semigroup itself.

<u>Definition</u>. A <u>binary operation</u> on a set S is a mapping from S S into S.

<u>Definition</u>. A binary operation Q is said to be <u>associative</u> if $Q(s_1,Q(s_2,s_3)) = Q(Q(s_1,s_2),s_3)$ for all $s_1,s_2,s_3 \in S$.

<u>Definition</u>. A binary operation Q is said to <u>commutative</u> if $Q(s_1,s_2) = Q(s_2,s_1)$ for all $s_1,s_2 \in S$.

<u>Definition</u>. A <u>semigroup</u> is a couple (S,Q) where S is a set on whose elements is defined an associative binary operation Q. The notation $Q(s_1,s_2) = s_3$ is somewhat cumbersome and will be used interchaneably with $(s_1 \cdot s_2) = s_3$ from this point on.

The <u>order</u> of a semigroup will mean the number of elements in the underlying set. For any given positive integer n there exists at least one semigroup of order n. For example, let $S = \langle 1, ..., n \rangle$ and define

$$Q(i,j) = \begin{cases} i+j & \text{if } i+j \leq n \\ i+j-n & \text{if } i+j > n \end{cases}$$

for i, j \in S. This is the cyclic semigroup on n elements.

In this paper we compute the distinct (up to isomorphism) binary operations on a set of finite order satisfying the above conditions. One way to specify a binary operation on a finite set is by means of the multiplication table.

Example. Let S = $\{1,2,3\}$ and define the binary operation Q by the following table.

The notation means that Q(1,1) = 1, Q(1,2) = 2, Q(1,3) = 3, Q(2,1) = 2, Q(2,2) = 3, and so on.

We use the following definitions as the basis for the construction of the isomorphism testing subroutine of the generation procedure.

Definition. Two semigroups (S_1,Q_1) and (S_2,Q_2) are called <u>isomorphic</u> if there exists a one-to-one mapping F of S_1 onto S_2 such that if $s_1,t_1 \in S_1$ and $F(s_1) = s_2$, $F(t_1) = t_2$ with $s_2,t_2 \in S_2$, then $F(Q_1(s_1,t_1)) = Q_2(s_2,t_2)$.

Definition. Two semigroups (S_1,Q_1) and (S_2,Q_2) are called <u>anti-isomorphic</u> if there exists a one-to-one mapping G of S_1 onto S_2 such that if $s_1,t_1 \in S_1$ and $G(s_1) = s_2$, $G(t_1) = t_2$ with $s_2,t_2 \in S_2$, then $G(Q_1(s_1,t_1)) = Q_2(t_2,s_2)$.

Given any semigroup (S,Q) we can in a natural way associate with it a semigroup (S^*,Q^*) defined by letting $S^* = S$, and for $s_1, s_2 \in S^*$ putting $Q^*(s_1, s_2) = Q(s_2, s_1)$. If G is the identity mapping on S, then $G(Q(s_1, s_2)) = Q^*(s_2, s_1)$ and hence (S,Q) is anti-isomorphic to (S^*, Q^*) .

If a semigroup (S,Q) is commutative, then it is both isomorphic and anti-isomorphic with (S^*,Q^*) since the identity map G is both an isomorphism and an anti-isomorphism. In the journal, <u>Mathematical Algorithms</u>, 1967, the editor remarked that the converse is also valid. However, the converse is not valid in general, as the following example illustrates.

Example. Let (S,Q) be defined by the table

then (S^*, Q^*) is defined by this table

	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	2
4	1	1	1	1

where the second table was determined from the first by $Q^*(s_1, s_2) = Q(s_2, s_1)$ with the only differences in the multiplication tables being $Q^*(3,4) = Q(4,3)$ and $Q^*(4,3) = Q(3,4)$. The anti-isomorphism G is the identity mapping, while the mapping F defined by F(1) = 1, F(2) = 2, F(3) = 4, and F(4) = 3 is an isomorphism linking (S,Q) with (S^*,Q^*) . It is interesting to note that orders two and three contain no non-commutative semigroup (S,Q) which is both isomorphic and anti-isomorphic to its (S^*,Q^*) , but that order four contains six such semigroups. By computing non-equivalent semigroups the question whether or not a given semigroup is isomorphic to its anti-isomorphic image is left unanswered

unless the semigroup in question is commutative. In computing semigroups distinct up to isomorphism we avoid this difficulty.

III. ALGORITHM FOR COMPUTING SEMIGROUPS OF FINITE ORDER

Using the fact that the only condition on the algebraic system under consideration in addition to closure is associativity, we are able to build a systematic generating procedure. Noting that in any given multiplication table if Q(i,j) = k, then Q(k,m) = Q(i,Q(j,m)), we construct a procedure to complete any partially completed table. In essence, we utilize the associative law to complete the unfilled portion of the table. The first step consists of placing a few key values in a blank multiplication table. Then as much as possible of the table is completed by the application of the above equation.

Example. Let S = $\{1, 2, 3, 4\}$ and let Q be partially defined by the following table.

Then by applying the above associativity equation to the table we find $(3\cdot1) = ((1\cdot2)\cdot1) = (1\cdot(2\cdot1)) = (1\cdot2) = 3$ and $(1\cdot3) = (1\cdot(1\cdot2)) =$ $((1\cdot1)\cdot2) = (1\cdot2) = 3$ as well as $(1\cdot4) = 4$, $(2\cdot4) = 4$, $(4\cdot1) = 4$, and $(4\cdot2) = 3$. We also have $(4\cdot3) = (3\cdot3)$ and $(4\cdot4) = (3\cdot4)$. The following table results.

Of course, in supplying the original six values a check must be made to determine whether or not they satisfy the associative law to make the completion of the table worthwhile. The table must be checked again when completed since not all associativity conditions need necessarily be used in the completion of it.

The above example points out the difficulties that arise when a partially completed table has values which are not conducive toward further generation. When this happens additional values must be supplied to restart the generation procedure. The choosing of these additional values must follow a pattern and must exhaust all possibilities. In the above example the values 1,2,3, and 4 must be tested in both the x and y positions, which results in a total of 16 additional cases to be checked.

In the case of order four we set initial values in six positions. We chose the positions (1,1), (1,2), (2,1), and (2,2) to facilitate the generation procedure by filling a fourth of the blank multiplication table, while we picked the two additional positions (2,3) and (3,2) to be used as launching points for the completion of the table. We chose an initial six positions for two reasons. First, since every finite semigroup has an idempotent element,¹ then every finite semigroup is isomorphic to some semigroup with the element one (1) in the position (1,1). If in a semigroup of order n the element i is idempotent, then define an isomorphism F such that F(i) = 1, F(1) = i, and F(k) = k for k = 2, ..., i - 1, i + 1, ..., n. The second reason is that the remaining five positions have to be filled in a manner that exhausts all possible combinations of the values one (1) through four (4). The number of initial cases, ranging from the values 1,1,1,1,1 to the values

¹Clifford, A. H. and Preston, G. B., <u>The Algebraic Theory of</u> Semigroups, v. 1, p. 20, American Mathematical Society, 1961.

4,4,4,4,4 in the five positions, is 4^5 , or 1024. The addition of any more initial positions would increase this number by a factor of four for each added position.

We developed this generating procedure because of time limitations on the use of the computer. For orders two and three the number of positions set with initial values were four and nine respectively. We exhaustively checked all possible combinations in these two orders since the number of cases to be checked was low. For order two there were only 2^4 cases while for order three there were only 3^9 cases. However, for order four there are 4^{16} , or over four billion, cases to be examined, which proved to be much too time consuming to allow the exhaustive procedure.

IV. IMPLEMENTATION OF ALGORITHM

We divided the algorithm into three basic parts in order to place it on the computer using the Fortran programming system. The first and primary part consists of producing the completed multiplication tables by application of the associative equation, Q(k,m) = Q(i,Q(j,m)) when Q(i,j) = k, to the initial values of the partially completed tables. The initial values are supplied by use of nested "DO loops." The number of nested "DO loops" used were four, nine, and five for orders two, three, and four respectively. Hence, for orders two and three entire multiplication tables were set with initial values and the algorithm degenerated into an exhaustive test of every possible multiplication table.

For order four we found that seven nested "DO loops" would be the maximum number for practical purposes, that is, any more would result in too much time consumption. We decided on five for the reasons stated before. After the initial values are supplied we check for violations of the associative law. If there are none the computer then applies the associative equation to the initial values and then to generated values until the generation procedure ceases. At this point the number of blanks remaining in the multiplication table determines to which further generation subroutine the computer switches. Once the multiplication table is completed we are finished with the first part of the algorithm.

The second part of the algorithm consists of the main associativity test. We check the entire multiplication table since the generation procedure does not necessarily use every associativity condition. In this associativity test as well as in previous ones we make use of the "LOGICAL IF" statement and self-subscripting capabilities of the Fortran language.

Example. IF (I(KK,I(LL,MM)).NE.I(I(KK,LL),MM)) GO TO 100 Once a multiplication table passes the second part it is given a number and recognized as the representation of a semigroup.

In the third part of the algorithm we take these multiplication tables and determine those which are not isomorphic to any of the others. This select group then represents a collection of semigroups which are distinct up to isomorphism.

As we mentioned before, additional subroutines were added to produce all groups as well as semigroups of the specified finite orders. These subroutines follow the third part of the algorithm.

We include the programs used and the output obtained in the latter part of this paper.

TA	BL	E	Ι

ORDER	1	2	3	4
Number of semigroups distinct up to isomorphism	1	5	24	188
Number of commut a tive semigroup s	1	3	12	58
Number of non-commutative isomorphic anti-isomorphic semigroups	0	0	0	6
Number of groups distinct up to isomorphism	1	1	1	2

SEMIGROUP	S	۰F	CRD	ER TH	IC DIS	TINCT	UP	TC I	SCMCRPH	ISM
1	1 SE	MIG	FCUP	N U M E I S	SER IS CCMMLT	ATIVE				
1	1 SEE SE SE			NUME IS HAS HAS HAS	SER IS COMMUT LEFT RIGHT IDENT	ATIVE ² IDENT IDEN ITY	ΙΤΥ ΤΙΤΥ	X = 2 Z =	2 SUCH 2 SUCH	тант тант
12	1 2 5 E 5 E	MIG MIG YZ MIG YZ	RCUP RCUP FY RCUP FY	NUME HAS HAS	BER IS RIGHT RIGHT	3 IDEN IDEN	ΤΙΤΥ ΤΙΤΥ	Z = Z =	1 SUCH 2 SUCH	ТНАТ ТНАТ
1	2 SE SE	MIG MIG XY MIG XY	RCUP RCUP = Y = Y = Y	NUME HAS HAS	BER IS LEFT LEFT	4 IDENT IDENT	ITY ITY	X = 1 X = 2	SUCH SUCH	ТНАТ ТНАТ
12	2 SEE SE	MIC MIC MIC MIC MIC MIC		NUMF IS HAS HAS IS	BER IS COMMUT LEFT RIGHT ICENT A GRC	ATIVE IDENT IDEN IDEN UP	ΙΤΥ ΤΙΤΥ	X = 1 Z =	SUCH 1 SUCH	ТНАТ ТНАТ

SEMIGROUPS OF CROER THREE DISTINCT UP TO ISCMORPHISM 1 1 1 1 1 1 1 1 SEMIGROUP NUMBER 1 SEMIGROUP IS COMMUTATIVE 1 1 1 1 1 1 1 1 2 SEMIGRCUP SEMIGRCUP (1,2) SEMIGRCUP 1 NUMBER 2 HAS A SUBSEMIGROUP OF ORDER TWO IS COMMUTATIVE 1 1 1 1 1 1

1	1 3						
-	SEMIGROUP	NUME	BER	3			
	SEMIGROUP	HAS	Α	SUBSEMIGROUP	CF	CRDER	TWO
	SEMIGROUP	HAS	A	SUBSEMIGROUP	CF	CRDER	TWO
	SEMIGRCUP	IS	COM	MUTATIVE			

1	1	1						
1	1	1						
1	2 SEN SEN	3 IGRCUP IGROUP (1-2)	NUMB HAS	ER	4 SUBSEMIGROU	IP OF	CRDER	TWO
	SEN	IGRCUP	HAS	A	SUBSEMIGROU	IP OF	CRDER	TWO
	SEN	I GRCUP SUCH	HAS THA	TE	FT IDENTITY KY=Y	× =	3	

1	1	1								
	SEMI SEMI SEMI	GPOUP GPOUP (1,2) GPOUP (1,3)	NUMPE HAS HAS	A A	SUBS SUBS	EMIGR EMIGR	OUP CUP	CF CF	CRDER CRDER	TWO TWO
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1	1 1 2 1			
3	3 3 SEMIGROUP SEMIGROUP (1,2) SEMIGROUP (1,3) SEMIGROUP SUCH	NUMBER 9 HAS A SUBSEMIGROUP HAS A SUBSEMIGROUP HAS RIGHT IDENTITY THAT YZ=Y	CF ORDER CF CRDER Z = 2	TWO TWO
1 1 1	1 1 2 2 SEMIGROUP SEMIGROUP	NUMBER 10 HAS A SUBSEMIGROUP	CF CRDER	тио
	(1,2) SEMIGPOUP (2,3) SEMIGRCUP	HAS A SUBSEMIGROUP IS CCMMUTATIVE	CF CRDER	TWO
1	1 1			
1	2 2			
1	2 3 SEMIGRCUP SEMIGRCUP (1,2)	NUMBER 11 HAS A SUBSEMIGROUP	CF ORDER	TWO
	SEMIGROUP (1,3) SEMIGROUP (2,3) SEMIGROUP	HAS A SUBSEMIGROUP IS COMMUTATIVE	CF CRDER	TWO
	SEMIGROUP SEMIGROUP	THAT XY=Y HAS RIGHT IDENTITY THAT YZ=Y HAS IDENTITY	Z = 3	
1	1 1			
1	2 2			
1	3 3 SEMIGROUP SEMIGROUP (1,2) SEMIGROUP (1,3) SEMIGROUP (2,3) SEMIGROUP SUCH	NUMBER 12 HAS A SUBSEMIGROUP HAS A SUBSEMIGROUP HAS A SUBSEMIGROUP HAS RIGHT IDENTITY THAT YZ=Y	OF ORDER OF CRDER CF CRDER Z = 2	TWO TWO TWO
	SEMIGREUP	THAT YZ=Y	(= 3	

1	1	1								
1	2	3								
1	2 SEMI SEMI	3 IGPCUP IGPOUP	NUMBE HAS	R	13 SUBS	SEMI	GRCUP	CF	CRDER	TWO
	SEM	IGPOUP	HAS	А	SUBS	SEMI	GROUP	OF	CRDER	TWO
	SEMI	IGROUP	HAS	Δ	SUBS	SEMI	GRCUP	CF	CRDER	TWC
	SEM	IGFOUP	HAS	LEE		IDEN	TITY	X =	2	
	SEMI	SUCH SUCH	HAS	LEÂ	(Y = Y (Y = Y	I D E N	ΤΙΤΥ	Χ =	3	
1	1	1								
1	2	3								
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1	1	1								
1	2	3								
3	3 SEMI	3 IGRCUP IGPCUP (1,2)	NUMBE	R	15 SUBS	SEMI	GROUP	CF	CRDER	TWO
	SEMI	(1,3)	HAS	Δ	SUBS	SEMI	GPCUP	CF	CRDER	TWO
	SEM	(2,3)	HAS	A	SUBS	SEMI(GROUP	LF	CRDER	IWO
	SEM]	SUCH	THAT	LEF	$(\mathbf{Y} = \mathbf{Y})$	UEN	1 I J Y	X =	2	
	SEMI	SUCH	THAT	KIG Y	7=Y	TDE	N L I Y	Ζ =	. 2	
	S. 14 19 1	unn Um	1.40	11.0	. 1 1 1 1 1					

1	1	1								
2	2	2								
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1 1 1	1 1 SEMIC SEMIC SEMIC	3 3 6 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	NUMBE H∆S HAS	R A	17 SURS SUBS	EMI	GROUP GRCUP	C F C F	GRDER CRDER	тис тис
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1	2 3		
1	3 SEMICPCUP SEMICRCUP (1,2) SEMIGRCUP (1,3) SEMIGPCUP SEMIGPCUP SEMIGPCUP SEMIGPCUP	NUMBER 20 HAS A SUBSEMIGROUP OF CROER HAS A SUBSEMIGROUP OF CROER HAS A SUBSEMIGROUP OF CROER HAS LEFT IDENTITY X = 2 THAT XY=Y HAS RIGHT IDENTITY Z = 2 THAT YZ=Y HAS IDENTITY	TWO TWO TWO
1	1 3		
3	3 1		
	SEMIGROUP	HAS A SUBSEMIGROUP OF CRDER	TWO
	SE'41 GRCUP (1.3)	HAS A SUBSEMIGROUP OF CROER	TWO
	SEMIGROUP	IS COMMUTATIVE HAS LEFT IDENTITY X = 2	
	SUCH SEMIGROUP SUCH	HAI XY=Y HAS PIGHT IDENTITY Z = 2 THAT YZ=Y	
	SEMIGROUP	HAS IDENTITY	
1	2 2		
2	1 1		
2	1 1 SEMIGROUP SEMIGROUP (1,2) SEMIGROUP	NUMBER 22 HAS A SUBSEMIGRCUP OF CROER IS COMMUTATIVE	TWO

1	2 2					
1	2 3					
1	2 SEMIGEOUP SEMIGEOUP (1,2) SEMIGEOUP (1,3) SEMIGEOUP (2,3) SEMIGEOUP SUCH SEMIGEOUP	NUMBER HAS A HAS A HAS A HAS LEF THAT X HAS LEF THAT X	23 SUBSEMIGROUP SUBSEMIGROUP SUBSEMIGROUP T IDENTITY Y=Y T IDENTITY Y=Y	CF CF CF X = X =	CRDER CRDER CRDER 1 2	T W O T W O T W O
	SUCH	THAT	Y = Y		-	

1 2 3 2 3 1 3 ¹ ² ^{SEMIGFCUP} NUMBER ²⁴ ^{SEMIGFCUP} IS COMMUTATIVE ^{SEMIGPCUP} HAS LEFT IDENTITY X = 1 ^{SUCH} THAT XY=Y ^{SEMIGFCUP} HAS PIGHT IDENTITY Z = 1 ^{SUCH} THAT YZ=Y ^{SEMIGFCUP} HAS IDENTITY ^{SEMIGFCUP} HAS IDENTITY ^{SEMIGFCUP} IS 4 GPCUP

1	1	1	1	
1	1	1	1	
1	1	1	1	
1	1	1	1	
	SEN	AIGROUP	NUMBER 1 IS COMMUTATI	VE
1	1	1	1	
1	1	1	1	
1	1	1	1	
1	1		2 NUMBED 2	
	SEA	ATCECUE	IS COMMUTATI	VE
1	1	1	1	
1	1	1	1	
1	1	1	1	
1	1		4 NUMBER 3	
		ALC S GID	IS COMMUTATI	VE
1	1	1	1	
1	1	1	1	
1	1	1	1	
1	1	ALCECHE	1 NUMBER 4	
1	1	1	1	
1	1	1	1	
1	1	1	1	
1	1			
	-1-1	TURNER	140100. P	

SEMIGROUPS OF CROER FOUR DISTINCT, UP TO ISOMORPHISM

1	1	1	1		
1.	1	1	1		
1	1	1	2		
1	CENIC CENIC	2 PCUP RCUP	2 NUMBEP IS COMMUTA	12 1 V F	
1	1	1	1		
1	1	1	1		
1	1	1	2		
1	1 SEWIC	2 R 011F R 011F	3 NUMBER IS COMMUTA	13 TIVE	
1	1	1	1		
1	1	1	1		
1	1	1	3		
1	1	1	4		
	_ CENIC	₽Û1₽	NUMBER	14	
1	1	1	1		
1	1	1	1		
1	1	1	3		
1	1 SEWIG	3 R () U P R () I P	4 NUMBER IS COMMUTA	15 TIVE	
1	1	1	1		
1	1	1	1		
1	1	1	3		
1	2 SEMIC	POUP	4 NUMBED	16	
1	1	1	1		
1	1	1	1		
1	1	1	3		
1	2 SEMIG	3 RCUP RCUP RCUP	4 NUMBEP HAS LEFT THAT XY=Y	17 LDENTITY X	= 4

1	1	1	1	
1	1	1	1	
1	1	2	1	
1	1 SEMI(SEMI(1 GRCUP GRCUP	4 NUMBER IS COMMUT	29 ATIVE
1	1	1	1	
1	1	1	1	
1	1	2	1	
1	1 SEMI	2 GREUP	2 NUMBER	30
1	1	1	1	
1	1	1	1	
1	1	2	1	
4	SEMI	4 GRCUP	4 NUMBER	31
1	1	1	1	
1	1	1	1	
1	1	2	2	
1	1 SEMIO SEMIO	2 GROUP GROUP	2 NUMBER IS COMMUT	ATIVE
1	1	1	1	
1	1	1	2	
1	1	2	3	
1	2 SEMI SEMI SEMI SEMI	3 GROUP GROUP GROUP SUCH GROUP GROUP	4 NUMBER IS COMMUT HAS LEFT THAT XY= HAS RIGHT THAT YZ= HAS IDENT	$\begin{array}{c} 33\\ \text{ATIVE}\\ \text{IDENTITY} X = 4\\ \text{Y}\\ \text{IDENTITY} Z = 4\\ \text{Y}\\ \text{ITY} \end{array}$
1	1	1	4	
1	1	1	4	
1	1	2	4	
1	1 SEMI	1 GROUP	4 NUMBER	34

1	1	1	1		
1	1	1	1		
1	1	3	3		
1	1 Semi Semi	3 GPCUP GPCUP	4 NUMBER IS CCMMU	TATIVE	
1	1	1	1		
1	1	1	1		
1	1	3	3		
1	1 Semi	GREUP	4 NUMBER	42	
1	1	1	1		
1	1	1	1		
1	1	3	3		
1	2 SEMI SFMI	3 GROUP GROUP SUCH	4 NUMBER HAS LEFT THAT XY	43 IDENTITY	X = 4
1	1	1	1		
1	1	1	1		
1	1	3	4		
1	1 SEMI	GROUP	4 NUMBER	44	
1	1	1	1		
1	1	1	1		
1	1	3	4		
1	1 SEMI SEMI	4 GROUP GROUP	3 NUMBER IS COMMU	45 TATIVE	
1	1	1	1		
1	1	1	1		
1	1	3	4		
4	4 SEMT	4	4	1.6	

1	1	1	1	
1	1	1	2	
1	1	3	1	
1	1 Semic	1 GRCUP	4 NUMBER	47
1	1	1	1	
1	1	1	2	
1	1	3	1	
1	2 SEMIC SEMIC	1 PCUP PCUP	NUMBER IS COMMUTA	ATIVE
1	1	1	1	
1	1	1	2	
1	1	3	3	
1	1 SEMIC SEMIC	3 SPCUP SPCUP SUCH	4 NUMBER HAS RIGHT THAT YZ=	49 IDENTITY Z = 4 Y
1	1	1	1	
1	1	1 -	2	
1	1	3	3	
1	2 SEMIC SEMIC SEMIC SEMIC	3 GROUP GROUP SPCUP SUCH SPCUP SUCH GPCUP	4 NUMBER IS COMMUTA HAS LEFT THAT XY=Y HAS RIGHT HAS RIGHT HAS IDENT	50 ATIVE IDENTITY X = 4 Y IDENTITY Z = 4 Y ITY
1	1	1	4	
1	1	1	4	
1	1	3	4	
1	1 Semi(1 GRCUP	4 NUMBER	51
1	1	1	4	
1	1	1	4	
1	1	3	4	
1	1 SEMIC	4 GPIGUP	4 NUMBER	52

1	1	1	4		
1	1	1	4		
1	1	3	4		
4	4 SEN SEN	4 I G R CUP I G R CUP	1 NUMBER IS COMM	UTATIVE	
1	1	1	4		
1	1	1	4		
1	1	3	4		
4	4 SEM SEM	4 IGROUP IGROUP	4 NUMBER IS COMM	UTATIVE	
1	1	1	1		
1	1	1	1		
1	2	3	1		
4	4 Sem	4 'IGROUP	4 NUMBER	55	
1	1	1	1		
1	1	1	1		
1	2	3	3		
1	² SEM	3 IGROUP	3 NUMBER	56	
1	1	1	1		
1	1	1	1		
1	2	3	3		
1	2 SEM SEM	3 IGPOUP IGROUP SUCH	4 NUMBER HAS LEF THAT X	T IDENTITY Y=Y	X = 4
1	1	1	1		
1	1	1	1		
1	2	3	3		
1	2 SEM	4 IGROUP	4 NUMBER	58	
1	1	1	1		
---	------------------------------	---------------------------	---	----------------------------	----------------
1	1	1	1		
1	2	3	4		
1	2 SEMIG SEMIG SEMIG		4 NUMBER HAS LEFT THAT XY=1 HAS LEFT THAT XY=1	59 IDENTITY IDENTITY	X = 3 X = 4
1	1	1	1		
1	1	1	1		
1	2	3	4		
1	2 SEMIG SEMIG	4 RCUP FCUP SUCH	3 NUMBER HAS LEFT THAT XY=1	60 IDENTITY Y	X = 3
1	1	1	1		
-	1	1	1		
1	2	- 3	4		
4	4 SEMIG SEMIG	4 ROUP ROUP SUCH	4 NUMBER HAS LEFT THAT XY=1	1 IDENTITY	X = 3
1	1	1	1		
1	1	1	2		
1	2	3	1		
1	1 Semig	1 ROUP	4 NUMBER	62	
1	1	,	3		
1	1	1	1		
1	2	2	2		
1	2	2	2		
T	SEMIG	ROUP	NUMBER	63	

1	1	1	1
1	1	1	2
1	2	3	3
1	2 SEMIG SEMIG SEMIG	3 ROUP SUCH ROUP SUCH ROUP	4 NUMBER 64 HAS LEFT IDENTITY $X = 4$ THAT $XY=Y$ HAS PIGHT IDENTITY $Z = 4$ THAT $YZ=Y$ HAS IDENTITY
1	1	1	4
1	1	1	4
1	2	3	<u> </u>
1	1 SEMIG	1 RCUP RCUP SUCH	4 NUMBER 65 HAS LEFT IDENTITY X = 3 THAT XY=Y
1	1	1	4
1	1	1	۷
1	2	3	۷
1	SEMIG	4 ROUP ROUF SUCH	A NUMBER 66 HAS LEFT IDENTITY X = 3 THAT XY=Y
1	1	1	4
1	1	1	4
1	2	3	4
4	4 SEMIG SEMIG	4 RCUP RCUP SUCH	1 NUMBER 67 HAS LEFT IDENTITY X = 3 THAT XY=Y
1	1	1	4
1	1	1	4
1	2	3	4
4	4 SEMIG SEMIG	4 ROUP ROUP SUCH	4 NUMBER 68 HAS LEFT IDENTITY X = 3 THAT XY=Y

-

SEALCOUD NUMBER SEMIGROUP NUMBER L CENTGROUP NHMPER 1 1 4 SENIGROUP NUMBER 72 SENIGROUP HAS FIGHT IDENTITY 7 = 4 SUCH THAT Y7=Y $\frac{1}{2} = \frac{3}{2} = \frac{4}{2}$ $\frac{1}{2} = \frac{3}{2} = \frac{4}{2}$ $\frac{1}{2} = \frac{3}{2} = \frac{4}{2}$ $\frac{1}{2} = \frac{3}{2} = \frac{3}{2}$ $\frac{1}{2} = \frac{3}{2} = \frac{3}{2}$ $\frac{2}{2} \frac{1}{2} \frac{4}{2} \frac{4}{2} \frac{7}{2} \frac{7}$

~

1	1	1	1		
1	1	2	2		
1	1	3	3		
1	SEWIG CEMIC		4 NUMBED HAS PICHT THAT YZ=Y HAS PICHT THAT YZ=Y	IDENTITY	Z = 3 Z = 4
1	1	1	1		
1	1	2	2		
1]	3	3		
1	SEMIC SEMIC SEMIC SEMIC	3 ROUF ROUF SUCH ROUP ROUP	4 NUMBER HAS LEFT THAT XY=Y HAS RIGHT THAT Y7=Y HAS IDENTI	R2 IDENTITY IDENTITY	x = 4 7 = 4
1	1	1	1		
1	1	2	2		
1	1	3	4.		
1	1 Z⊏WIC	3 P CIJP	a Nijmre p	83	
1	1	1	1		
1	1	2	2		
1	1	3	L <u>e</u>		
1	SENIC SEMIC	4 RCUP RCUP SUCH	3 NUMBEP HAS PIGHT THAT Y7=Y	TDENTITY	<u>7</u> = 3
1	1	1	4		
1	1	2	٢		
1	1	3	4		
1	1 SEMIG	1 RCUP	4 NUMBER	85	

SEMIGROUP HAS RIGHT IDENTITY Z = 3 SUCH THAT YZ=Y L SEMIGROUP NUMBER 87 SEMIGROUP HAS FIGHT IDENTITY Z = 3 SUCH THAT Y7=Y SENTGROUP SEMIGROUP NUMBER 98 SEMIGROUP HAS RIGHT IDENTITY SUCH THAT YZ=Y ζ = 3 2 SEMIGROUP NUMBER IS COMMUTATIVE CEMIGREUP SEWIGROUP SEWIGROUP NUMBER 90 IS COMMUTATIVE HAS LEFT IDENTITY X = 4 THAT XY=Y HAS PIGHT IDENTITY Z = THAT YZ=Y HAS IDENTITY SEMIGROUP SEMIGROUP Z = 4

1	1	1	1		
1	1	2	2		
1	2	3	3		
1	2 SEMIC SEMIC SEMIC	4 SPCUP SPCUP SUCH SPCUP SUCH	4 NUMBER 91 HAS RIGHT ID THAT YZ=Y HAS RIGHT ID THAT YZ=Y	ENTITY	7 = 3 2 = 4
1	1				
1	1	1	1		
1	1	2	2		
1	2	3	4		
1	2 SEMIC SEMIC SEMIC	BRCUP SPOUP SUCH SRCUP SUCH	4 NUMBER 92 HAS LEFT IDE THAT XY=Y HAS LEFT IDE THAT XY=Y	NTITY 2	X = 3 X = 4
1	1	1	1		
1	1	2	2		
1	2	3	4		
1	2 SEMIC SEMIC SEMIC SEMIC	4 SPCUP SRCUP SRCUP SUCH SRCUP	3 NUMBER 93 IS COMMUTATIV HAS LEFT IDE THAT XY=Y HAS RIGHT ID THAT YZ=Y HAS IDENTITY	E NTITY) ENTITY	X = 3 Z = 3
1	1	1	4		
1	1	2	4		
1	2	3	4		
1	1 SEMIC SEMIC	1 PCUP SUCH	4 NUMBER 94 HAS LEFT IDE THAT XY=Y	NTITY)	X = 3
1	1	1	4		
1	1	2	4		
1	2	3	4		
1	1 SEMIC SEMIC	4 GREUP SUCH SREUP SUCH	4 NUMBER 95 HAS LEFT IDE THAT XY=Y HAS RIGHT ID THAT YZ=Y	NTITY >	(= 3 Z = 3
	SEMIG	FCUP	HAS ICENTITY		

1	1	1	4
1	1	2	4
1	2	3	4
4	4 SEMIC SEMIC SEMIC SEMIC	4 RCUP RCUP SPOUP SUCH SROUP SUCH GROUP	1 NUMBER 96 IS COMMUTATIVE HAS LEFT IDENTITY X = 3 THAT XY=Y HAS RIGHT IDENTITY Z = 3 THAT YZ=Y HAS IDENTITY
1	1	1	4
1	1	2	4
1	2	3	4
4	4 SEMIO SEMIO SEMIO SEMIO	4 GRCUP GRCUP SRCUP SUCH SRCUP SUCH GRCUP	4 NUMBER 97 IS COMMUTATIVE HAS LEFT IDENTITY X = 3 THAT XY=Y HAS RIGHT IDENTITY Z = 3 THAT YZ=Y HAS IDENTITY
1	1	1	1
1	2	1	1
1	1	3	1
1	1 SEMIO SEMIO	1 ROUP ROUP	4 NUMBER 98 IS COMMUTATIVE
1	1	1	1
1	2	1	1
1	1	3	1
4	4 SEMIC	4 GRCUP	4 NUMBER 99
1	1	1	1
1	2	1	1
1	1	3	3
1	1 SEMIC	3 BROUP BROUP	3 NUMBER 100 IS COMMUTATIVE

1	1	1	1	
1	2	1	1	
1	1	3	3	
1	1 Sem Sem	3 IGROUP IGROUP	4 NUMBER 101 IS COMMUTATIVE	
1	1	1	1	٠
1	2	1	1	
1	1	3	3	
1	¹ SEM	4 IGROUP	4 NUMBER 102	
1	1	1	1	
1	2	1	1	
1	1	3	4	
1	1 Sem	3 IIGROUP	4 NUMBER 103	
1	1	1	1	
1	2	1	1	
1	1	3	4	
1	1 Sem Sem	4 IGROUP IGROUP	3 NUMBER 104 IS COMMUTATIVE	
1	1	1	1	
1	2	1	1	
1	1	3	4	
4	4 Sem	4 IGROUP	4 NUMBER 105	
1	1	1	1	
1	2	1	2	
1	1	3	3	
1	2 SEM SEM	3 IIGROUP IIGROUP IIGROUP SUCH	4 NUMBER 106 IS COMMUTATIVE HAS LEFT IDENTITY THAT XY=Y	X = 4 7
	SEP	SUCH	HAS IDENTITY	

1	1	1	4		
1	2	1	4		
1	1	3	4		
1	1 SEMIG	1 RCUP	4 Number	107	
1	1	1	4		
1	2	1	4		
1	1	3	4		
1	1 SEMIG	2 PCUP	4 NUMBER	103	
1	1	1	4		
1	2	1	4		
1	1	3	4		
4	4 SEMIG SEMIG	4 POUP RCUP	1 NUMBER IS CCMMUT	109 FATIVE	
1	1	1	4		
1	2	1	4		
1	1	3	4		
4	4 SEMIG SEMIG	4 RCUP RCUP	4 NUMBER IS COMMUT	110 ATIVE	
1	1	1	1		
1	2	1	1		
3	3	3	3		
3	³ SEMIG	3 FCUP	4 NUMBER	111	
1	1	1	1		
1	2	1	1		
3	3	3	3		
4	4 SEMIG SEMIG	4 RCUP RCIJP SUCH	4 NUMBER HAS RIGH1 THAT YZ=	112 I IDENTITY	2 = 2

1	1	1	1
1	2	1	2
3	3	3	3
1	2 SEMI	1 GROUP	2 NUMBER 113
1	1	1	1
1	2	1	2
3	3	3	3
1	2 SEMIO SEMIO	1 GROUP GROUP SUCH	4 NUMBER 114 HAS RIGHT IDENTITY Z = 4 THAT YZ=Y
1	1	1	1
1	2	ĩ	2
3	3	3	3
1	2 SEMI SEMI SEMI	3 GROUP GROUP SUCH GROUP SUCH GROUP	4 NUMBER 115 HAS LEFT IDENTITY X = 4 THAT XY=Y HAS RIGHT IDENTITY Z = 4 THAT YZ=Y HAS IDENTITY
1	1	1	1
1	2	1	2
3	3	3	3
1	4 SEMI SEMI	1 GROUP GROUP SUCH GROUP SUCH	4 NUMBER 116 HAS RIGHT IDENTITY Z = 2 THAT YZ=Y HAS RIGHT IDENTITY Z = 4 THAT YZ=Y
1	1	1	1
1	2	1	2
3	3	3	3
3	4 SEMIO SEMIO	3 GROUP GROUP SUCH GROUP SUCH	4 NUMBER 117 HAS RIGHT IDENTITY Z = 2 THAT YZ=Y HAS RIGHT IDENTITY Z = 4 THAT YZ=Y

1	1	1	1	
1	2	1	4	
3	3	3	3	
1	2 SEMI	1 GROUP	4 NUMBER 118	
1	1	1	1	
1	2	1	Δ4	
3	3	3	3	
1	4 SEMI SEMI	1 GROUP GROUP SUCH	2 NUMBER 119 HAS RIGHT IDENTITY Z = 2 THAT YZ=Y	2
1	1	1	1	
1	2	1	4	
3	3	3	3	
4	4 SEMI SEMI	4 GRCUP GRCUP SUCH	4 NUMBER 120 HAS RIGHT IDENTITY Z = 2 THAT YZ=Y	2
1	1	1	4	
1	2	1	4	
3	3	3	4	
4	4 SEMI SEMI	4 GRCUP GROUP SUCH	4 NUMBER 121 HAS RIGHT IDENTITY Z = 2 THAT YZ=Y	2
1	1	1	1	
1	2	2	4	
1	2	2	4	
1	2 SEMI	2 GP OUP	4 NUMBER 122	
1	1	1	1	
1	2	2	4	
1	2	2	4	
1	4 SEMI	4 GR CUP GR CUP	2 NUMBER 123 IS COMMUTATIVE	

1	1	1	1		
1	2	2	4		
1	2	2	4		
1	4 SEMIC SEMIC	4 ROUP RCUP	4 NUMBER IS COMMUT	124 ATIVE	
			1		•
1	1	1	1		
1	2	2	4		
1	4	2	4		
~	SEMIC	GROUP	NUMBER	125	
1	1	1	4		
1	2	2	4		
1	2	2	4		
1	1 SEMTO			126	
	JET: A		NOTDER	120	
1	1	1	4		
1	- 2	2	4		
1	2	2	4		
1	4	4	4		
	SEMIC	GROUP	NUMBER	127	
1	1	1	4		
1	2	2	4		
1	2	2	4		
4	4 SEMIO	A GROUP	1 NUMBER	128	
	SEMIC	GROUP	IS COMMUT	ATIVE	
1	1	1	1		
1	2	2	2		
1	2	3	5		
1	SEMIC	ROUP	NUMBER	129	
	SEMIC	GROUP	HAS LEFT	IDENTITY	X = 4
	SEMIC	GROUP	HAS RIGHT	IDENTITY	Z = 4
	SEMIC	GROUP	HAS IDENT	ÎTY	

2 4 4 SEMIGRCUP NUMBER 130 SEMIGRCUP HAS PIGHT IDENTITY SUCH THAT Y7=Y SUCH THAT Y7=Y 2 = 3 7 = 4SEMIGROUP SEMIGROUP SUCH SEMIGROUP NUMBER 131 SEMIGROUP HAS LEFT IDENTITY SUCH THAT XY=Y SEMIGROUP HAS LEFT IDENTITY SUCH THAT XY=Y X = 3 $\chi = 4$ 2 SEMIGPCUP NUMBER 132 IS COMMUTATIVE HAS LEFT IDENTITY X = 3 THAT XY=Y HAS RIGHT IDENTITY Z = 3 THAT Y7=Y HAS IDENTITY SEMIGRCIJP SEMIGROUP SEMIGROUP SUCH SUCH CEMIGROUP NUMBER 133 HAS LEFT IDENTITY THAT XY=Y HAS FIGHT IDENTITY THAT Y7=Y HAS IDENTITY SEWIGDCUD SEMIGROUS X = 3SUCH SEWICEUP SOCH 7 = 3 2 2 SEMIGRCUP SEMIGROUP NUMBER 134 HAS LEFT IDENTITY NUMBER X = 3

1	1	1	1	
1	2	2	4	
1	2	3	4	
1	2 SEMIC SEMIC SEMIC	4 SROUP SUCH SROUP SUCH SPOUP	4 NUMBER 135 HAS LEFT IDENTITY THAT XY=Y HAS RIGHT IDENTITY THAT YZ=Y HAS IDENTITY	X = 3 Z = 3
1	1	1	1	
1	2	2	4	
1	2	3	4	
1	CEMIC CEMIC CEMIC CEMIC		2 NUMBER 136 IS COMMUTATIVE HAS LEFT IDENTITY THAT XY=Y HAS RIGHT IDENTITY THAT Y7=Y HAS IDENTITY	X = 3 Z = 3
1	1	1	1	
1	2	2	4	
1	2	3	4	
۷	ZEWIC ZEWIC ZEWIC	4 SRCUP SPCUP SUCH SRCUP SRCUP	4 NUMBER 137 HAS LEFT IDENTITY THAT XY=Y HAS RIGHT IDENTITY THAT YZ=Y HAS IDENTITY	X = 3 Z = 3
1	1	1	4	
1	2	2	4	
1	2	3	4	
1	1 SEMIC	1 SROUP SROUP SUCH	4 NUMBER 138 H ⁴ S LEFT IDENTITY THAT XY=Y	X = 3
1	1	1	4	
1	2	2		
1	2	3	4	
1	J Sewic Sewic Sewic		4 NUMBER 139 HAS LEFT IDENTITY THAT XY=Y HAS PIGHT IDENTITY THAT YZ=Y HAS IDENTITY	X = 3 Z = 3

L. $\begin{array}{c} 1 \\ \text{NUMBEP} & 141 \\ \text{IS} & \text{COMMUTATIVE} \\ \text{HAS} & \text{LEFT} & \text{IDENTITY} & X = 3 \\ \text{THAT} & XY = Y \\ \text{HAS} & \text{FIGHT} & \text{IDENTITY} & \overline{Z} = 3 \\ \text{THAT} & Y7 = Y \\ \text{HAS} & \text{IDENTITY} \end{array}$ L SEMIGROUP SEMIGROUP SEMIGROUP SUCH A NUMBER 142 HAS RIGHT IDENTITY THAT YZ=Y HAS PIGHT IDENTITY THAT YZ=Y HAS PIGHT IDENTITY THAT YZ=Y SEMIGRAUE SEWIGROUP SUCH 7 = 2SEMIGROUP 7 = 3 SEMIGROUP 7 = 4SUCH NUMBER 143 HAS PIGHT IDENTITY THAT Y7=Y HAS RIGHT IDENTITY THAT YZ=Y SEMIGROUP SEMIGROUP SUCH SEMIGROUP 7 = 2Z = 3SUCH SEMIGROUP NUMBER

1	1	1	4	
1	2	2	۷,	
1	3	3	4	
1	ZEMIC ZEMIC ZEMIC	4 ROUF ROUF SUCH ROUP SUCH	Δ NUMBER 145 HΔS PIGHT IDENTITY THΔT YZ=Y HΔS PIGHT IDENTITY THΔT YZ=Y	? = 2 Z = 3
1	1	1	4	
1	2	2	4	
1	3	3	4	
٤	2 CENIC CENIC		1 NUMBEP 146 HAS PIGHT IDENTITY THAT YZ=Y HAS PIGHT IDENTITY THAT YZ=Y	7 = 2 Z = 3
1	1	1	1	
1	2	3	4,	
1	2	3	4	
1	SEMIC SEMIC SEMIC SEMIC	3 ROUR SUCH ROUR SUCH ROUR SUCH	A NUMBEP 147 HAS LEFT IDENTITY THAT XY=Y HAS LEFT IDENTITY THAT XY=Y HAS LEFT IDENTITY THAT XY=Y	X = 2 X = 3 X = 4
1	1	1	1	
1	2	3	4	
1	2	3	۷	

1. Ŀ NUMBER 150 HAS LEFT IDENTITY THAT XY=Y HAS LEFT IDENTITY THAT XY=Y SEMIGROUP CENTGROUP X = 2 SEMIGROUP SUCH X = 3SEMIGROUP NUMBER 151 SEMIGROUP HAS LEFT IDENTITY SUCH THAT XY=Y SEMIGROUP HAS LEFT IDENTITY SUCH THAT XY=Y X = 2X = 33 SEMIGROUP NUMBER 152 IS COMMUTATIVE CENTGOQUE Ŀ NUMBER 153 HAS LEFT IDENTITY X = 2 THAT XY=Y HAS RIGHT IDENTITY Z = 2 THAT Y7=Y HAS IDENTITY SEWIGROUP SEMIGROUP SUCH

1	1	1	L .
1	2	3	L;
1	3	2	۵
1	1 СЕМІ СЕМІ		A NUMBEO 154 HAS LEFT IDENTITY X = 2 THAT XY=Y
1	1	1	۷
1	2	3	1.
1	3	2	۵.
1			A NUMBER 155 HAS LEFT IDENTITY X = 2 THAT XY=Y HAS PIGHT IDENTITY Z = 2 THAT Y7=Y HAS IDENTITY
1	1	1	4
1	2	3	4
1	3	2	4
۷	семі семі семі		1 NUMBER 156 IS COMMUTATIVE HAS LEFT IDENTITY X = 2 THAT XY=Y HAS FIGHT IDENTITY Z = 2
	CE M I	GROUP	HAS IDENTITY
1	1	1	1
1	2	3	٤.
3	3	3	3
3			2 NUMBER 157 HAS LEFT IDENTITY $X = 2$ THAT $XY=Y$ HAS RIGHT TOENTITY $Z = 2$ THAT $Y7=Y$ HAS IDENTITY

1	1	1	1		
1	2	3	4		
3	3	3	3		
۷	4 SEMIG SEMIG	4 RCUP RCUP SUCH RCUP RCUP	4 NUMBER HAS LEFT THAT XY= HAS RIGHT THAT Y7= HAS IDENT	158 IDENTITY IDENTITY TITY	X = 2 Z = 2
1	1	1	1		
2	2	2	2		
3	3	3	3		
۷	4 SEWIG SEWIG	4 RCUP RCUP RCUP RCUP SUCH RCUP SUCH	4 NUMBE P HAS PIGHT THAT Y7= HAS PIGHT THAT Y7= HAS PIGHT THAT Y7= HAS PIGHT THAT Y7=	150 IDENTITY IDENTITY IDENTITY IDENTITY IDENTITY Y	z = 1 z = 2 z = 3 z = 4
1	1	3	3		
1	1	3	3		
1	1	3	3		
1	SEMIG	3 RCUP	3 NUMBER	160	
1	1	3	3		
1	1	3	3		
1	1	3	3		
1	1 SEMIG	3 P C U P	4 NIJMBER	161	
1	1	3	3		
1	1	3	3		
1	1	3	3		
1	SEWIC SEWIC	3 PCUP RCUF SUCH	4 NUMBER HAS LEFT THAT XY=	162 IDENTITY	X = 4.

1	1	3	4				
1	1	3	4				
1	1	3	L				
1	1 - E M J C	9 2 3	4 NUMBER	163			
1	1	3	4				
1	1	3	L <u>:</u>				
1	1	3	<i>t</i> .				
1	3 CEWIG	P Ū I O	4 NUMRER	164			
1	1	3	3				
1	1	3	3				
3	3	1	1				
<u>;</u>	3 SEWIG	1 RCUP PCUP	1 NUMBER IS COMMUT	165 ATIVE			
1	1	3	3				
1	1	3	3				
3	3	1	1				
3	SEMIC SEMIC	1 RCUP RCUP	2 NUMBEP IS COMMUT	166 ATIVE			
1	1	3	3				
1	2	3	3				
1	1	3	3				
1	1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	s CNs 3	4 NUMBER	157			
1	1	3	3				
1	2	3	<u>د</u>				
1	1	3	3				
3	SEWIC SEWIC		4 NUMBER HAS LEFT THAT XY= HAS LEFT THAT YY=	168 IDENTITY IDENTITY	× ×	11	2 4

1	1	3	4	
1	2	3	4	
1	1	3	4	
1	I SEMIC	3 ROUP ROUP SUCH	4 NUMBER 169 HAS LEFT IDENTITY X THAT XY=Y	= 2
1	1	3	4	
1	2	3	4	
1	1	3	4	
1	4 SEMIC	3 RCUP RCUP SUCH	4 NUMBER 170 HAS LEFT IDENTITY X THAT XY=Y	= 2
1	1	3	3	
1	2	3	4	
1	3	3	1	
1	4 SEMIG SEMIG SEMIG	3 ROUP SUCH SUCH SUCH SUCH SROUP	2 NUMBER 171 HAS LEFT IDENTITY X THAT XY=Y HAS PIGHT IDENTITY Z THAT YZ=Y HAS IDENTITY	= 2 = 2
1	1	3	4	
1	2	3	4	
1	3	3	4	
1	4 SEMIG SEMIG		4 NUMBER 172 HAS LEFT IDENTITY X THAT XY=Y HAS RIGHT IDENTITY Z THAT YZ=Y HAS IDENTITY	= 2 = 2
1	1	3	3	
1	2	3	3	
3	3	1	1	
3	3 SENIG	1 RCUP RCUP	1 NUMBER 173 IS COMMUTATIVE	

1	1	3	3
1	2	3	3
3	3	1	1
3	SENIC SENIC	1 RCHP RCHP SUCH	1 NUMBER 174 HAS PIGHT IDENTITY Z = 2 THAT YZ=Y
1	1	3	3
1	2	3	٤
3	3	1	1
3	3 SENIC		1 NUMBER 175 HAS LEFT IDENTITY X = 2 THAT XY=Y
1	1	3	3
1	2	3	1.
3	3	1	1
3	A SEMIC SEMIC SEMIC SEMIC	1 ROUP ROUP ROUP ROUP ROUP ROUP	1 NUMBER 176 IS COMMUTATIVE HAS LEFT IDENTITY $X = 2$ THAT XY=Y HAS FIGHT IDENTITY $Z = 2$ THAT Y7=Y HAS IDENTITY
1	1	3	3
1	2	3	4
3	3	1	1
3	C C C C C C C C C C C C C C C C C C C		2 NUMBER 177 IS COMMUTATIVE HAS LEFT IDENTITY X = 2 THAT XY=Y HAS SIGHT IDENTITY Z = 2 THAT YZ=Y HAS IDENTITY
1	2	2	2
2	2	2	
2	1	1	1
2	1	1	1
2	SENIC	S CUE	NUMBER 178 IS COMMUTATIVE

1	2	3	4	
1	2	3	4	
1	2	3	4	
1	2 SEMIG SEMIG SEMIG SEMIG	3 ROUP SUCH ROUP SUCH SUCH SUCH SUCH	4 NUMBER 179 HAS LEFT IDENTITY THAT XY=Y HAS LEFT IDENTITY THAT XY=Y HAS LEFT IDENTITY THAT XY=Y HAS LEFT IDENTITY THAT XY=Y	X = 1 X = 2 X = 3 X = 4
1	2	3	1	
2	3	1	2	
3	1	2	3	
1	2 SEMIG	3 RCUP RCUC	1 NUMBER 180 IS COMMUTATIVE	
1	2	3	1	
2	3	1	2	
3	1	2	3	
1	2 SEMIG SEMIG SEMIG SEMIG	3 ROUP ROUP SUCH RCUP SUCH ROUP	4 NUMBER 181 IS COMMUTATIVE HAS LEFT IDENTITY THAT XY=Y HAS RIGHT IDENTITY THAT YZ=Y HAS IDENTITY	X = 4 Z = 4
1	2	3	4	
2	3	1	4	
3	1	2	4	
4	4 SEMIG SEMIG SEMIG	4 ROUP ROUP SUCH ROUP	4 NUMBER 182 IS COMMUTATIVE HAS LEFT IDENTITY THAT XY=Y HAS RIGHT IDENTITY	X = 1 Z = 1
	SEMIG	ROUP	HAS IDENTITY	

1	2	3	4
1	2	3	۷.
3	٢.	1	2
3	SEMIC SEMIC SEMIC	1 RCUP SUCH RCUP SUCH	2 NUMBER 183 HAS LEFT IDENTITY X = 1 THAT XY=Y HAS LEFT IDENTITY X = 2 THAT XY=Y
1	2	2	4
2	4	4	1
2	4	4	1
۷	SEWIC SEWIC		2 NUMBEP 184 IS COMMUTATIVE
1	1	4	۵
1	1	4	4
2	2	3	3
2	2 SEMIC	3 R C I J C	3 NUMBER 185
1	1	4	4
2	2	3	3
3	3	2	2
۷	CENIC SEMIC CEMIC	1 RCUP RCUP SUCH RCUP SUCH	1 NUMBEP 186 H \leq PIGHT IDENTITY Z = 1 TH \leq TYZ=Y H \leq PIGHT IDENTITY Z = 2 TH \leq TYZ=Y
1	2	3	4
2	1	4	3
3	4	1	2
۷	3 SEWIG SEWIG SEWIG	2 PCUP RCUP RCUP RCUP RCUP RCUP RCUP RCUP	$\begin{array}{c} 1 \\ NUMBEP & 1P7 \\ IS COMMUTATIVE \\ HAS LEFT IDENTITY X = 1 \\ THAT XY=Y \\ HAS PIGHT IDENTITY 7 = 1 \\ THAT Y7=Y \\ HAS IDENTITY \\ IS A GROUP \end{array}$

1	2	3	4
2	1	4	3
3	4	2	1
4	3 SEMIO SEMIO SEMIO SEMIO		2 NUMBER 198 IS COMMUTATIVE HAS LEET IDENTITY $X = 1$ THAT $XY=Y$ HAS RIGHT IDENTITY $Z = 1$ THAT $Y7=Y$ HAS IDENTITY IS A GROUP

SEM IGR T	CUPS C HEIP A	NF CRDE	R FOUR	WHICH IMAGE	ARE BUT	ISOMCRPHIC TO NOT COMMUTATIVE
1	1	1	1			
1	1	1	1			
1	1	1	1			
1	1 Sem	2 HIGRCUP	1 NUMBER	1		
1	1	1	1			
1	1	1	1			
1	1	2	1			
1	1 SEM	2 NIGPOUP	2 NUMBER	2		
1	1	1	1			
1	1	1	1			
1	1	1	3			
1	2 SE*	IGROUP	4 NUMBER	3		
1	1	1	1			
1	1	1	2			
1	2	3	1			
1	1 SEM	1 IGROUP	4 NUMBER	4		
1	1	1	1			
1	1	1	2			
1	2	3	2			
1	1 SE	IGRCUP	4 NUMBER	5		
1	1	4	4			
1	1	4	4			
2	2	3	3			
2	2 SEM	3 AIGRCUP	3 NUMBER	6		

```
INTEGER
                                      H
           DIMENSION I(10,10),M(20,3,3),L(5,5),LP(5,5),LF(5)
           N=2
H=1
           M6=0
                                   I 1 = 1 , N
J 1 = 1 , N
K 1 = 1 , N
           DČ
                       10
                       10
10
           DC
           DC
DC
           DC 10 [
I(1,1)=I1
I(1,2)=J1
I(2,1)=K1
I(2,2)=L1
L4=1
                                    I \bar{I} = \bar{I} \cdot N
           N4=1
15=1
15=0
           N5=0
           DO 15 I2=
K2=N+1
K2N=(N*N)+N
                                   I2=1,N
           K2N=(N*N)+N

DC 15 J2=K2,K2N

IF (J2.EQ.N+1) [

IF (J2.EQ.N+2) [

IF (J2.EQ.N+3) [

IF (J2.EQ.N+3) [

IF (J2.EQ.N+4) [

I(I2,J2)=I(I2,L2) [

I(J2,I2)=I(L2,I2) [

DC 20 M3=1,N

J3=(M3*N)+N
                                                              L2=I1
L2=J1
L2=K1
L2=L1
  15
            J3N = (M3 * N) + N
          J3N=(M3*N)+N

DC 20 K3=J3,J3N

DC 20 L3=1,N

I3=(((K3-(M3*N))*(N-L3))+((K3-(M3*N)+1)*L3))

IF (I(M3,I3).NE.I(K3,L3)) GC TO 24

CONTINUE
  20
        WRITE (6,21) H, ((I(J,K),K=1,2),J=1,2)
WRITE (7,210) ((I(J,K),K=1,2),J=1,2)
FORMAT (///1X'PERMUTATION NUMBER',I5,' IS
IGROUP',//2X,I2,4X,I2,//2X,I2,4X,I2)
FORMAT (///44X,//2X,I2,4X,I2,//2X,I2,4X,I2)
                                                                                                                                                                  SEMI
                                                                                                                                                        A
  21
210 FORMAT
          M6=M6+1
DC 16 16=1,N
DC 16 J6=1,N
M(M6,I6,J6)=I(I6,J6)
WRITE (6,17) M6
WRITE (7,17) M6
WRITE (7,17) M6
FORMAT (10X'SEMIGROUP
FORMAT (10X'SEMIGROUP
           M6 = M6 + 1
  16
  17
                                                                                     NUMBER
                                                                                                             IS ', I5)
           IF (M6
LF(1)=2
                                                                                 19
           LF(2) = 1
DC 18
                                   N6=1,N
L6=1,N
K6=1,N
                       18
18
18
           ĎÖ
           DC
            ĪF
                        (M(M6,L6,K6).EQ.N6)
                                                                                       L(L6,K6) = LF(N6)
          CONTINUE
LP(1,1)=L(2,2)
LP(1,2)=L(2,1)
LP(2,1)=L(1,2)
LP(2,2)=L(1,1)
L7=1
  18
```

GENERATION 0 F CRDER TWC SEMIGROUPS

GC

TO

13

12 12

I7=1,N

12 J7=1,N (LP(17,J7).NE.M(L7,17,J7))

11

DC

DG IF

12 CONTINUE GC TO L7=L7+1 ŤO 14 13 (L7.EQ.M6) GC ĨĖ TC 19 TO 11 RITE (6,E) L7 RMAT (10X'SEMIGROUP NUMBER 1,15) 14 WRITE 8 FERMAT IS **ISOMORPHIC** TC SEMIGRCUP 1 то 24 GC 22 I4=1,N 22 J4=1,N (I(I4,J4).NE.I(J4,I4)) ĎČ 19 DC GC TO 26 CONTINUE WEITE (22 (6,23) (7,23) (10X'SEMIGRCUP WRITE FORMAT IS 23 CCMNUTATIVE!) 27 K4=1,N (I(L4,K4).NE.K4) 26 GC TO 29 CUNTINUE 27 (6,28) (7,28) WPITE 14 Ē4 15=14 (10X'SEMIGROUP THAT XY=Y') 28 FORMAT LEFT IDENTITY X = 1, 12,HAS 11 SUCH 29 L4 = L4 + 129 L2=L4+1 IF (L4.GT.N) GO T GC TO 26 30 DC 31 M4=1,N IF (I(M4,N4).NE.M4) 31 CONTINUE WPITE (6,32) N4 WFITE (7,32) N4 NE-N6 TO 30 G C. TO 33 WFITE (0,32) N4 N5=N4 32 FORMAT (10X'SEMIGRCUP 1, SUCH THAT YZ=Y') 33 N4=N4+1 CT N) GD TO RIGHT IDENTITY Z = 1, 12HAS (N4.GT.N) TO 30 IF GO TC 34 ĜĊ GC TO 30 IF ((N5.NE.L5).CR.((N5.EQ.O).AND.(L5.EQ.O))) GC TO 24 WFITE (6,35) WRITE (7,35) FORMAT (10X'SEMIGROUP HAS IDENTITY') DC 37 J5=1,N IF (I(I5,J5).EC.N5) GC TO 38 CONTINUE GC TO 24 I5=I5+1 IF (J5.GT.N) GO TO 39 IF 1GC 34 35 36 37 38 (I5.GT.N) GO TO TO 36 ITE (6,40) ITE (7,40) RMAT (IOX'SFMIGRCUP ÎĒ GC 30 WFITE 39 40 24 10 FORMAT IS Δ GRCUP!) H=H+1 CONTINUE STOP 68 END

. .

```
INTEGEP H
DIMENSION
                                                           H
                                                                      I(15,15), M(400,5,5), LP(5,5), L(5,5), LF(5)
             ,LR(5)
N=3
H=1
          1
              ME=0
              MC = 0
              I1=1,N
I2=1,N
I3=1,N
                                   10
                                   10
10
              10
                                                       I4=1,N
I5=1,N
I6=1,N
                                  10
10
10
10
               DĈ
                                                        17=1,N
               DŌ
              DC
                                                       I8 = 1, N
           DC 10 18=1,N

DO 10 I9=1,N

U4=1

N4=1

L5=0

N5=0

J6=1

I(1,1)=I1

I(1,2)=I2

I(1,3)=I3

I(2,1)=I4

I(2,2)=I5

I(3,2)=I6

I(3,1)=I7

I(3,2)=I8

I(3,3)=I9

DC 15 J1=1,N

J2=N+1

J2N=(N*N)+N

DC 15 J3=J2,J2N

IF (J3 EQ 0N+1) J4=I1

IF (J3 EQ 0N+2) J4=I2

IF (J3 EQ 0N+3) J4=I3

IF (J3 EQ 0N+4) J4=I4

IF (J3 EQ 0N+4) J4=I5

IF (J3 EQ 0N+6) J4=I5

IF (J3 EQ 0N+6) J4=I5

IF (J3 EQ 0N+7) J4=I7

IF (J3 EQ 0N+7) J4=I7

IF (J3 EQ 0N+7) J4=I7

IF (J3 EQ 0N+7) J4=I9

I(J1,J3)=I(J1,J4)

I(J3,J1)=I(J4,J1)

DC 20 M1=1,N

M2=(M1*N)+1

M2N=(M1*N)+N

DC 20 M4=1,N

M5=(((M3-(M1*N))*(N-M4))+((M3-(M1*N)+1)*M4))

IF (I(M1,M5) NE I(M3,M4)) GC TO 24

CONTINUE

WEITE (6,21) H,((I(J,K),K=1,3),J=1,3)

ECOMME
                                                       19=1,N
               \bar{1}4 = 1
15
20 CONTINUE
        D CONTINUE
WRITE (6,21) H,((I(J,K),K=1,3),J=1,3)
FORMAT (///IX'PERMUTATION NUMBER',I5,' IS A SEMI
1 GROUP',//2X,I2,4X,I2,4X,I2,//2X,I2,4X,I2,4X,I2,//2X,
2I2,4X,I2,4X,I2)
M9=M9+1
M8=M8+1
D0 50 I10=1,N
D0 50 J10=1,N
D0 50 J10=1,N
M(M9,I10,J10)=I(I10,J10)
WRITE (6,51) M9
WRITE (7,51) M8
 21
 50
```

GENERATION

0F

CRDER

THREE

SEMIGROUPS

FCRMAT (10X'SEMIGROUP NUMBER' IF ((M9.E0.1).CR.(M9.E0.400)) DC 90 I20=1,N DC 90 I21=1,N DC 90 I22=1,N NUMBER 1, 15) 51 ĜŪ TC 5 DC 90 I22=1,N LF(1)=I20 LF(2)=I21 LF(3)=I22 LF(I20)=1 LF(I21)=2 LF(I22)=3 IF ((I20.EQ.I21).GR.(I20.EQ.I22).CR.(I21.EQ.I22)) GO TO 90 FO K10=1.N 1 80 80 DC K10=1, N 74 80 M10=1,N 80 M10=1,N (I(M10,N10).EQ.K10) Dr DC. IF (I(M10,N10).EQ.K10 CCNTINUE LF(1,1)=L(LR(1),LR(1)) LF(1,2)=L(LR(1),LR(2)) LF(1,3)=L(LR(1),LR(3)) LF(2,1)=L(LR(2),LR(1)) LF(2,2)=L(LR(2),LR(2)) LF(2,3)=L(LR(2),LR(3)) LF(3,1)=L(LR(3),LR(3)) LF(3,3)=L(LR(3),LR(3)) LF(3,3)=L(LR(3),LR(3)) L20=1 DC 60 I11=1,N DC 60 J11=1,N L(M10, N10) = LF(K10)IF 80 79 59 TC 61 GO 60 61 IZO=L20+1 IF (L20.E0.M9) GC T GC TO 59 WRITE (6,66) L20 FCRMAT (10X'SEMIGROUP NUMBER',15) TO 90 65 66 IS ISOMORPHIC TO SEMIGROUP 1 M8=M8-1 GC TO GC TO CONTINUE DC 500 10 GC 10 10 CONTINUE DC 500 I500=1,N J500=N+1-I500 DC 501 I501=1,N DC 501 I502=1,N IF ((I501.EQ.J500).OR.(I502.EQ.J500)) GC IF (I(I501.F02).EQ.J500) GO TO 500 CONTINUE N501=1 N5C2=2 N503=3 IF (J500.EQ.2) GC TC 503 IF (J500.EQ.2) GC TC 503 IF (J500.EQ.2) GC TC 503 GC TO 500 WFITE (7,502) N502,N503 GC TO 500 WFITE (7,502) N5C1,N503 GC TO 500 WFITE (7,502) N5C1,N502 FOPMAT (ICX'SEMIGROUP HAS A SUBSEMIGRCUP CRDEF TWO',//I4X'(',I1,',I1,')') CONTINUE 90 TC 501 501 503 504 G(10 2000) N5C1,N502 WFITE (7,502) N5C1,N502 FOPMAT (1CX'SEMIGROUP HAS CRDEF TWO',//14X'(',I1, CONTINUE DO 22 K1=1,N DC 22 K2=1,N IF (I(K1,K2).NE.I(K2,K1)) CONTINUE 505 502 0F 1 500 5 DC IF (I(K1,F2, CCNTINUE WFITE (6,23) FCEMAT (10X'SEMIGROUP DC 27 K4=1,N DC 27 K4=1,N IF (I(L4,K4).NE.K4) IF (I(L4,K4).NE.K4) GC TO 26 22 23 26 ΙS CCMMUTATIVE!) GC TO 29 27

```
WRITE (6,28)
WRITE (7,28)
                               - Ł4
                                 Ē4
     15=14
28 FCPMAT
                   (10X'SEMIGRCUP HAS
SUCH THAT XY=Y')
                                                           LEFT IDENTITY X = 1, I2,
    1//14X*
29
     L4=L4+1
    GC TO 26
DC 31 M6=1,N
IF (I(M6,N4).NE.M6)
CONTINUE
WEITE
            (L<sup>4</sup>.GT.N) GO TO
TO 26
                                                30
3 C
                                              GC
                                                     ΤO
                                                             33
31
     WFITE
               (6,32)
(7,32)
                                 N4
                                 N4
32 FORMAT
                   (10X 'SEMIGRCUP HAS
SUCH THAT YZ=Y')
                                                           RIGHT
                                                                      ICENTITY Z = 1, I2,
1//14X
33 N4=N4+1
         (N<sup>2</sup> • GT • N )
TO 30
     IF
                               GC
                                        TO
                                                34
     ĞC
     GL TO 30

IF ((N5.NE.L5).OR.((N5.EQ.O).AND.(L5.EQ.O)))

GO TO 24

WRITE (6.35)

FORMAT (IOX'SEMIGROUP HAS IDENTITY')

DC 37 J5=1.N

IF (I(J6.J5).EG.N5) GC TO 38
34 IF
   1
35 FCRM
36 DC
37 CONTINUE
GC TO 24
     J = J + 1
38
     IF (J5.GT.N) GC TC 39
GC TO 36
WRITE (6,40)
FORMAT (IOX'SEMIGROUP IS A
39
40 FOPMA
24 H=H+1
                                                               GRCUP!)
    IF (H.GT.2000C)
CONTINUE
                                      GO
                                            TC
                                                    63
10
     WPITE (6,25) H
FORMAT (///5X'H
STOP
END
68
25
                                      =',15)
```

	INTEGE4
	H = 1 $N_1 = 4$
	N = 1 N = 1
	$\begin{array}{c} I \\ I \\ 2 \\ 4 \\ - \\ 9 \\ 1 \\ 2 \\ 1 \\ - \\ 6 \end{array}$
	1 4 1 = 0 1 4 1 = 0
	$\begin{array}{c} 1 & 2 = c \\ 1 & 4 & 3 = c \end{array}$
	I < 4 = c C = 600 = K = 1 + 9
	$\frac{DC}{L(K3,K4)=9}$
600	
	DC 100 112=1,N
	DC 100 122=1.N
	DC 100 132=1,N
	I (1, 1) = I I I I (1, 2) = I I 2
	$\frac{1}{1} (\frac{1}{2}, \frac{3}{2}) = \frac{1}{1} \frac{1}{4}$
	! (2,1)=121 ! (2,2)=122
	I (2,3)=I23 I (2,4)=I24
	I(3,1)=I31 I(3,2)=I32
	I(3,3) = I33 I(3,4) = I34
	I(4, 1) = I41 I(4, 2) = I42
	I(4,2) = I42 I(4,2) = I42
	$ \begin{bmatrix} \Gamma & P \\ C \\$
	CC = POO = L3 = 1.3
	$IF (I(L1,I(L2,L3)) \cdot NE \cdot I(I(L1,L2),L3)) GC TC 8C1$
301	[IF] ((I(L1,L2,L3)).EG.9).OR.(I(I(L1,L2),L3).E0.9))
]	L GC TE 800 GC TE 100
300	
	WFITE $(6+16)$ L7+ $((I(K+L)+L=1+4)+K=1+4)$ N7=0
	NC=C N12=C
300	№13=C DC 500 КК=1-14
00	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
500	J(KK, K1, K2) = I(K1, K2)
5.0	IF (N7.EQ.D) GC TC 999

	I(3,3) = N7
	J(1,3,3)=N7 IF (N12.GE.1) GC TC 852
808	DC 850 LL=1,N DC 850 MM=1.N
	ĬF (J(1,LL,MM).GE.5) GO TO 851
95 1	N13=N13+1
850	CONTINUE
852	GC TO 902 N12=N12+1
	IF (N12.GE.5) GC TC 853
	I(LL,MM) = N12
853	N13=0
	N12=0 GC TO 905
902	J(13,I(1,3),3)=I(1,I(3,3)) J(13,I23,3)=I(2,I(3,3))
	J(13, I(3, 3), 3) = I(3, I(3, 3))
	J(14,3,I(3,1))=I(I(3,3),1)
	J(14,3,152) = I(1(3,31,2)) J(14,3,1(3,4)) = I(I(3,3),4)
	J(14,3,1(3,3))=I(I(3,3),3) IF (N7.NE.4) GO TO 999
	J(12,4,1) = I(3,I(3,1)) J(12,4,2) = I(3,I(3,2))
	J(12,4,3) = I(3,I(3,3)) $I(12,4,3) = I(3,I(3,3))$
	J(12,3,4) = I(I(3,3),3)
	J(12,1,4) = I(I(1,3),3)
	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
999 19	IF (I12-3)19,20,21 GD TO 22
2C	J(1, I12, 1) = I(1, I21) I(1, I12, 3) = I(1, I23)
	J(1, I12, 4) = I(1, I(2, 4))
	J(2,3,112) = I(I(3,1),2)
21	J(1,4,112) = I(1(4,1),2) J(1,112,1) = I(1,121)
	J(1, I12, 2) = I(1, I22) J(2, I12, 3) = I(1, I23)
	J(1, 12, 4) = I(1, 1(2, 4))
	J(1,2,112) = I(121,2)
	J(2, 4, 112) = I(I(4, 1), 2)
	GC TO 3C
22	IF (122-3123,24,25 GG TO 26
24	J(2, I22, 1) = I(2, I21) J(3, I22, 3) = I(2, I23)
	J(3, I22, 4) = I(2, I(2, 4)) $J(2, 1, I22) = I(I12, 2)$
	J(4,3,122) = I(132,2)
25	J(2, I22, I) = I(2, I21)
	J(4 + I 22 + 3) = I(2 + I 23)
	J(2, 1, 122, 4) = I(2, 1(2, 4)) J(2, 1, 122) = I(112, 2)
	J(2,2,122)=I(I22,2) J(4,3,I22)=I(I32,2)
	J(4,4,122) = I(I(4,2),2) N2=4

26 27	GO TO 30 IF(121-3)27,28,29 GC TO 31
28	J(3, I21, 1) = I(2, I11) J(5, I21, 3) = I(2, I(1, 3)) J(5, I21, 4) = I(2, I(1, 4)) J(3, 1, I21) = I(I12, 1) I(6, 3, I21) = I(I12, 1)
29	J(5,4,121) = I(I(4,2),1) J(3,121,1) = I(2,111) J(3,121,2) = I(2,112) I(4,121,2) = I(2,112)
	J(5, I21, 4) = I(2, I(1, 4)) J(3, 1, I21) = I(I12, 1) J(3, 2, I21) = I(I22, 1)
	J(6,3,121) = I(132,1) J(6,4,121) = I(I(4,2),1) N2=6 GC = TC = 30
31 32 33	IF(I23-3)32,33,34 GC TO 35 J(4,I23,1)=I(2,I(3,1)) J(7,I23,3)=I(2,I(3,3))
	J(7, I23, 4) = I(2, I(3, 4)) J(4, 1, I23) = I(I12, 3) J(8, 3, I23) = I(I32, 3) I(7, 4, I23) = I(I32, 3)
34	J(4, I23, 1) = I(2, I(3, 1)) J(4, I23, 2) = I(2, I32) J(8, I23, 3) = I(2, I(3, 3))
	J(4,1,1,23) = I(1,2,3) J(4,2,1,23) = I(1,2,3) J(4,2,1,23) = I(1,2,3) J(4,2,1,23) = I(1,2,3) J(8,3,1,23) = I(1,32,3)
3C	J(8,4,I23)=I(I(4,2),3) N2=8 N3=N2/2 D0 300 L3=N3,N2
300	DD 300 L4=1,N DD 300 L5=1,N IF (J(L3,L4,L5).NE.9) I(L4,L5)=J(L3,L4,L5)
35	IF (N2-EQ-8) GO TO 35 IF (N2-4)22,26,31 IF (I32-3)36,37,38
37	J(5, I32, 1) = I(3, I21) J(9, I32, 3) = I(3, I23) J(9, I32, 4) = I(3, I(2, 4))
38	J(5,1,132) = I(I(1,3),2) J(10,3,132) = I(I(3,3),2) J(9,4,132) = I(I(4,3),2) J(5,132,1) = I(3,121)
	J(5, I32, 2) = I(3, I22) J(10, I32, 3) = I(3, I23) J(9, I32, 4) = I(3, I(2, 4)) J(5, I32, 4) = I(3, I(2, 4))
	J(5,2,I32) = I(I23,2) J(10,3,I32) = I(I(3,3),2) J(10,4,I32) = I(I(4,3),2)
39	N2=10 N3=N2/2 IF (N3-EQ-0) GO TO 18 DO 301 L3=N3,N2
301	UU 301 L4=1,N DC 301 L5=1,N IF (J(L3,L4,L5).NE.9) I(L4,L5)=J(L3,L4,L5) CONTINUE
18 40	IF (I12.EQ.1) GC TC 40 IF (I12.EQ.2) GC TC 41 J(6,I12,3)=I(1,I23) J(6,I12,4)=I(1,I(2,4))

	J(6,3,112) = I(I(3,1),2)	`
41	J(6,4,112) = I(I(4,1),2) J(6,112,4) = I(1,I(2,4))	
	N6=6	
51	IF (I21.EQ.1) GC TO	42
42	J(7, I21, 3) = I(2, I(1, 3))	-+ C +-
	J(7,3,121) = I(132,1)	
43	J(7, I21, 4) = I(2, I(1, 4))	
	$N_{e=7}$	
52	IF (122-E0-1) GC TO	44
44	J(8, I22, 3) = I(2, I23)	
	J(8,3,122) = I(132,2)	
45	J(8, I22, 4) = I(2, I(2, 4))	
	J(8,4,122)=1(1(4,21,2) N6=8	
54	IF (123-EQ-1) GC TO	46
46	J(9, I23, 3) = I(2, I(3, 3))	41
	J(9,3,I23) = I(2,I(3,4)) J(9,3,I23) = I(I32,3)	
47	J(9, 4, 123) = I(I(4, 2), 3) J(9, I23, 4) = I(2, I(3, 4))	
	J(9,4,123) = I(I(4,2),3) N6=9	
55	IF (I32.E0.1) GC TO	48
48	J(10, 132, 3) = I(3, 123)	49
	J(10, 132, 4) = I(3, 1(2, 4)) J(10, 3, 132) = I(I(3, 3), 2)	
49	J(10, 4, 132) = I(1(4, 3), 2) J(10, 132, 4) = I(3, I(2, 4))	
	J(10,4,I32)=I(I(4,3),2) N6=10	
50	DD 302 L4=1, N DD 302 L5=1, N	
302	IF (J(N6,L4,L5).NE.9) CCNTINUE	1 (L4,L5)=J(N0,L4,L5)
53	IF (N6-7)51,52,53 IF (N6-9)54,55,56	
56	DU 400 L4=1,N DC 400 L5=1,N	
	$\begin{array}{c} IF & (I(L4,L5),EQ,4) & GC \\ IF & (I(L4,L5),EQ,3) & GO \end{array}$	TO 401 TO 402
401	$ \begin{array}{cccc} GC & TD & 400 \\ J(11,4,1) = I(L4,I(L5,1)) \end{array} $	
	J(11,4,2) = I(L4,I(L5,2)) J(11,4,3) = I(L4,I(L5,3))	
	J(11,4,4) = I(L4,I(L5,4)) J(11,3,4) = I(I(3,L4),L5)	
	J(11,2,4) = I(I(2,L4),L5) J(11,1,4) = I(I(1,L4),L5)	
	IF(J(11,4,4).EQ.9) J(1) GC TO 400	1,4,4)=I(I(4,L4),L5)
402	J(15,3,1) = I(L4,I(L5,1)) J(15,3,2) = I(L4,I(L5,2))	
	J(15,3,4) = I(L4, I(L5,4)) J(15,1,3) = I(I(1,L4),L5)	
	J(15,2,3)=I(I(2,L4),L5) J(15,4,3)=I(I(4,L4),L5)	
	J(15,3,3) = I(I(3,14),15)	
```
IF (J(15,3,3).EQ.9) J(15,3,3)=I(L4,I(L5,3))
400 CONTINUE
      no
                   L4=1,N
             8
       DŌ
                   L5=1,N
              8
       N4=2
      IF (J(1,L4,L5)-9)8,10,10
J(1,L4,L5)=J(N4,L4,L5)
IF (J(N4,L4,L5)-9)8,11,11
  10
  11 N4=N4+1
           (N4.EQ.13) GC TC
(N4.EQ.14) GC TC
TO 10
(J(13.L4,L5).EQ.9)
TC 10
       IF
                                                 14
       ĪĒ
                                                 8
       GŪ
                                                 GO
                                                        TO
                                                               13
  14
      IF
       ŝΓ
      J(1,L4,L5)=I(L4,L5)
CONTINUE
  13
WFITE (6,105)
105 FORMAT (20X,15)
                                   N7
       LE=100*H
       WPITE
GC T
                  (6,16) L8, ((J(1,K,L),L=1,4),K=1,4)
              ŤΟ
              (N7-1)904,9C3,9C3
(J(1,3,3).GE.5)
T0_909
905
       TF
       ĪF
                                            GC
                                                  TO
904
                                                          903
       GP
      IF
              (N12.NE.0)
903
                                  GC
                                          TC
                                                 900
       N7 = N7 + 1
     JU T

L4=1,N

IF (J(1,L4,L5).NE.9)

IF (N5-2)5,6,7

J(1,L4,L5)=5

N5=N5+1

GC TO 9

J(1,L4,15

N5=N5
                                               102
909
                                                 GC
                                                        TO
                                                               15
      J(1,L4,L5)=6
N5=N5+1
GC TO 9
   6
      IF (N5-4)12,4,3
J(1,L4,L5)=7
N5=N5+1
GC TO 9
J(1-L6)
  12
      J(1,L4,L5)=15
N5=N5+1
GC TO 9
   4
      IF (N5-6)2,1,9
J(1,L4,L5)=16
N5=N5+1
GC TO
    3
   2
      J(1, L4, L5) = 17
    1
                     õ
       GC
              TO
      CONTINUE
CONTINUE
DC 200
DC 200
  15
     DC 200 L1=1,N
DC 200 L2=1,N
IF (J(1,L1,L2).EQ.9)
GC TO 200
J(1,L1,L2)=I(L1,L2)
CCNTINUE
IF (N9 E0
                                               GO TO
                                                             201
201
200
```

	I (2, 1) = I21 $I (2, 2) = I22$ $I (2, 3) = I23$ $I (2, 4) = I24$ $I (3, 1) = I31$ $I (3, 2) = I32$ $I (3, 3) = I33$ $I (3, 4) = I34$ $I (4, 1) = I41$ $I (4, 2) = I42$ $I (4, 3) = I43$			
	I(4,4)=144 IF (N7.FQ.0) GC TO 905	GO	TO	707
7 C 7	NS=N9+1 IF (N9.GE.2) GD TO 905	GO	то	102
102	CONTINUE H=H+1			
100 101	IF (H.EQ.50) CONTINUE CONTINUE STOP	GO	TO	101
	END			

INTEGER H DIMENSION I(70,7C),J(70,70),J1(5,5,5) N=4H=1 DC 10 K9=1,200 FEAD (5,11) M,((I(L,K),K=1,4),L=1,4) FOFMAT (17I4) IF (M.EQ.9999) GD TO 10 11 N12=0 DC 60 LL=1,N DC 60 KK=1,N IF (I(LL,KK).GE.5) GC TO 60 N12=N12+1 IF (N12.GE.6) GC CCNTINUE GC TO 61 61 TC 107 CONTINUE DO 100 I21=1, N DC 100 I22=1, N DC 100 I23=1, N DC 101 I24=1, N DC 101 I25=1, N J(I24, I25)=I(I24, I25) DC 50 I11=1, N DF (I(I11, I12) EQ.5) IF (I(I11, I12) EQ.6) IF (I(I11, I12) EQ.7) GC TO 45 J(I11, I12)=I21 60 101 51 52 53 102 GO TO ŤÕ GO ŤŇ **GO** J(I11,I12)=I21 GO TC 49 51 GO. J(I11, I12)=I22 GC TO 49 52 J(I11,I12)=I23 IF (J(I11,I12).LE.4) I(I11,I12)=J(I11,I12) GC TO 102 53 49 GO TO 50 GC TÓ CONTINUE 50 105 DC DC 333 DC 105 M3=1,N DC 105 M4=1,N J1(1,M3,M4)=J(M3,M4) CCNTINUE DC 12 K1=1,N DC 12 K2=1,N DC 12 K3=1,N IF (J(J(K1,K2),K3).NE.J(K1,J(K2,K3))) CCNTINUE IF (H.EQ.1) GC TO 103 DC 104 M1=1,N DC 104 M2=1,N IF (J1(2,M1,M2).NE.J1(1,M1,M2)) GC T CCNTINUE M3=1,N 105 GD TO 24 12 TO 103 CONTINUE 104 GC T WFITE WRITE WRITE ŤĊ 24 (ć,21) H,((7,21) H,((6,22) M (I15,16I2) (I60) H,((J(L,K),K=1,4),L=1,4) H,((J(L,K),K=1,4),L=1,4) 103 FORMAT 21 22 24 H=H+1106 DC M5=1,N CC 106 M6=1,N J1(2,M5,M6)=J1(1,M5,M6) CCNTINUE CCNTINUE 106 ĞČ ŤO 10

FOUR

SEMIGROUPS

PART

TWO

GENERATION

OF

ORDER

107	I(1,1)=1 DC 120 I21=1,N DD 120 I22=1,N DC 120 I23=1,N DC 120 I24=1,N DC 120 I25=1,N DC 120 I26=1,N DC 120 I26=1,N
109	DC 109 L9=1,N J(L8,L9)=I(L8,L9)
111	DC 500 III=1,N DC 500 II2=1,N IE (I(I))-112).80.5) CC TO 510
	IF (I(III,II2).EQ.6) GD TO 520 IF (I(III,II2).EQ.7) GC TO 530 IF (I(III,II2).EQ.15) GC TO 690 IF (I(III,II2).EQ.16) GC TO 700 IF (I(III,II2).EQ.17) GC TO 710
510	J(I11,I12)=I21
52C	J(I11,I12) = I22
53C	GO TO 490 J(I11,I12)=I23
690	GC TO 490 J(I11,I12)=I24
70C	GC TO 490 J(I11,I12)=I25
71C	GC TO 490 J(I11,I12)=I26
490	IF (J(I11,I12).LE.4) GO TO 500 I(I11,I12)=J(I11,I12)
500	GC TO 111 CONTINUE
125	DC 125 M4=1,N J1(1,M3,M4)=J(M3,M4) CONTINUE DC 82 K1=1,N
	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
82	CONTINUE
	$\begin{array}{c} 1 \\ 1 \\ 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\$
	IF (J1(2,M1,M2).NE.J1(1,M1,M2)) GO TO 123
124	CONTINUE GO TO 94
123	WRITE (6,91) H, ((J(L,K),K=1,4),L=1,4) WRITE (7.91) H. ((J(L,K),K=1,4),L=1,4)
91 92 94	WRITE (6,92) M FORMAT (115,1612) FORMAT (160) H=H+1
	DC 126 M5=1,N DC 126 M6=1.N
126	JI(2,M5,M6)=JI(1,M5,M6) CONTINUE CONTINUE
ĪČ	CONTINUE STOP END

INTEGER H DIMENSION I(20.20).M(900.5.5).IP(5.5).I(5.5).IE(5).IR
N=4 M9=0
M8=0 DC 100 K9=1,900
$ \begin{array}{c} L 4 = 1 \\ N 4 = 1 \\ N 4 = 1 \end{array} $
15 READ (5,20) + ((I(J,K),K=1,4),J=1,4)
20 FCPMAT (115,1612)
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
ĪĒ (Ī(I(KK,ĒĹ),MM).NE.I(KK,I(LL,MM))) GC TO 998 999 ČONTINUE
GC TC 996 998 M20=M9+1
997 FORMAT (30X NO , 19)
396 CENTINUE
$M_{8}=M_{8+1}$ $W_{8}=M_{8+1}$ $W_{8}=M_{8+1}$ $W_{8}=M_{8+1}$ $W_{8}=M_{8+1}$ $W_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8+1}$ $M_{8}=M_{8}$ M_{8} $M_{8}=M_{8}$ $M_{8}=M_{8}$ $M_{8}=M_{8}$ M_{8
WRITE (7,210) ((I(J,K),K=1,4),J=1,4) 210 FORMAT (///2X,I2,4X,I2,4X,I2,4X,I2,4X,I2,4X,I2,4X,I2,4X,I2
112,4X,12,4X,12,4X,12,//2X,12,4X,12,4X,12,4X,12) 21 FCRMAT (///1X PERMUTATION,19, IS A SEMIGROUP,//
14X,12,4X,12,//2X,12,4X,12,4X,12,4X,12,4X,12,//2X,12,4X,12,4X 22X,12,4X,12,4X,12,4X,12)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
WRITE (6,51) M8
51 FCPMAT (ICX'SEMIGROUP NUMBER', I7) IF (M9.EQ.1) GO TO 5
DC 90 I40=1,N DC 90 I41=1,N
DC 90 I42=1,N DC 90 I43=1,N
LF(1) = 140 LF(2) = 141 LF(2) = 141
LF(3)=142 LF(4)=143 LF(40)=1
LP(141) = 2 LP(142) = 3
IR(I43)=4 IF ((I41.EQ.I42).OR.(I41.EQ.I43).OR.(I42.EQ.I43)) GO
IF ((I40.EQ.I41).OR.(I40.EQ.I42).OR.(I40.EQ.I43)) GO DO 80 K10=1,N
$\begin{array}{cccc} DU & EO & MIO=1, N \\ DC & BO & NIO=1, N \\ C & C & MIO=1, N \\$
$\frac{1}{30} = \frac{1}{100} = \frac{1}{$
LP(1,2) = L(LR(1), LR(2)) $LP(1,3) = L(LR(1), LR(3))$
LP(1,4) = L(LR(1), LR(4))

GENERATION OF ORDER FOUR SEMIGROUPS PART THREE

LF(2,1)=L(LR(2),LR(1)) LP(2,2)=L(LR(2),LR(2)) LP(2,3)=L(LR(2),LR(3)) LF(2,4)=L(LR(2),LR(3)) LF(2,4)=L(LR(2),LR(3)) LF(2,4)=L(LR(2),LR(4)) LP(3,1)=L(LR(3),LR(1)) LP(3,2)=L(LR(3),LR(2)) LP(3,3)=L(LR(3),LR(3)) LF(2,4)=L(LR(3),LR(4)) LF(4,1)=L(LR(4),LR(1)) LF(4,2)=L(LR(4),LR(2)) LF(4,3)=L(LR(4),LR(3)) LP(4,4)=L(LR(4),LR(3)) LP(4,4)=L(LR(4),LR(4)) LO=1 DC 60 I31=1 * 79 ĖŚ 60 J11=1,N (LP(I31,J11).NE.M(L20,I31,J11)) ĐĒ ĪF GO TC 61 GC TC 6 L2C=L2O+1 IF (L2O GC TC 5 WRITE (6 6 C 61 (L20.EQ.M8) TC 59 90 GC TC (6,66) L2C (IOX'SEMIGROUP 65 FOFMAT 66 IS ISOMORPHIC TC SEMIGRCUP M8 = M8 - 1GC TO CONTINUE CONTINUE DC 500 DC 500 100 90 5 DC 500 I 500=1,N K500=N+1-I500 L500=N+1-J500 N501=1 N502=2 N503=3 N504=4 IF (I500-F0.15 N504=4 IF (I500.EQ.J500) GC TO 501 DC 502 I502=1,N DC 502 J502=1,N IF ((I502.EQ.K500).OR.(I502.EQ.L500).OR.(J502.EQ.K500) 1).OR.(J502.EQ.K500).OR.(I(I502,J502).EQ.K500)) IF ((I(I502,J502).EQ.K500).OR.(I(I502,J502).EQ.L500)) 1 GO TO 500 CONTINUE 502 M500=K500+L500 (M500 EQ • 3) (M500 EQ • 3) (M500 EQ • 4) (M500 EQ • 5) (M500 EQ • 6) (M500 EQ • 6) 510 511 512 513 ĪĒ GC TC ĪF GO TO GO ĪF TO ĪF TO (M500.6,503) E (6,503) N502,N504 TC 500 ((K500.60.1).0R.(L500.60.1)) E (6,503) N501,N504 TC 500 (6,503) N502,N503 ŤŌ 514 İF WFITE GO TO 510 WRITE GC TO 511 512 I F GC TO 515 WRITE GC TO 500 (6,503) N502,N503 500 (6,503) N501,N503 500 (6,503) N501,N502 (10X'SEMIGRCUP H TWO ',2I2) WRITE GC T 515 WPITE GC T 513 WFITE FORMAT 514 503 HAS SUBSEMIGROUP A OF A1 (10X SEPIGROUP HAS A 5005CH P TWC ',2I2) TC 50C 504 I504=1,N ((I504.EQ.K500).CR.(J504.EQ.K500)) (I(I504.J504).EQ.K500) GC TC 50 TNUE 10FDEP GC T DC DC IF 501 GC TO 504 F 500 I CCNTINUE IF (K500-EQ-1) IF (K500-EQ-2) IF (K500-EQ-3) 504 GC GC GC 506 507 TO TO 508

(K500.EQ.4) E (6,505) TD 500 IF GC TC 509 N502 . N503 . N504 WRITE GC TO 506 GU TO (6,505) N501,N503,N504 500 (6,505) N501,N502,N504 500 (6,505) N501,N502,N503 (10X'SEMIGROUP HAS A 507 WFITE 508 GC ŤΟ 509 505 WFITE FORMAT SUBSEMIGROUP OF 10RDER T CCNTINUE THREE 1,312) 500 DC 22 K1=1,N DC 22 K2=1,N IF (I(K1,K2).NE.I(K2,K1)) CONTINUE GC TO 26 22 (7,23) (6,23) (10X'SEMIGREUP WRITE WRITE FCRMAT (10X'SEMIGRC DC 27 K4=1,N IF (I(L4,K4).NE.K4) 23 26 IS CCMMUTATIVE!) GC TO 29 27 CONTINUE (7,28)(6,28)WPITE L4 WRITE 14 L5 = L4FORMAT (10X'SEMIGRCUP HAS LEFT. IDENTITY X =', 12,/ 28 THAT XY = Y'1 29 L4 = L4 + 1ĪF (L4.GT.N) GC TO 30 TO 26 31 M6=1,N (I(M6,N4).NE.M6) GO DC 30 IF 33 GC TO CONTINUE 31 (7, 32)(6, 32)WRITE **N4** N4 N5=N4 FORMAT (10X'SEMIGROUP YZ=Y') HAS RIGHT IDENTITY Z = 1, I2,32 1 33 N4 = N4 + 1IF (N4.GT.N) GO TO 34 TO 30 ((N5.NE.L5).OR.((N5.EQ.O).AND.(L5.EQ.O))) E (7,35) E (6,35) AT (10X'SEMIGRCUP HAS IDENTITY') 37 J5=1,N (I(J6,J5).EC.N5) GC TO 38 ĜD 34 TO GO WRITE WRITE FORMAT 35 DÖ 36 IF CONTINUE GC TO 37 24 38 JE = JE + 1(J6.GT.N) TO 36 ĬF GC GO TO 39 (6,40) (7,40) (10X'SEMIGROUP WFITE 39 FORMAT CONTINUE CONTINUE CONTINUE STOP 40 24 100 105 IS GRCUP!) 4 END

Clifford, A. H., and Preston, G. B., <u>The Algebraic Theory of Semigroups</u>, v.l, American Mathematical Society, 1961.

Ljapin, E. S., <u>Semigroups</u>, American Mathematical Society, 1963.

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In this paper an algorithm for comput discussed. A computation procedure is de finite order, all semigroups which are di restrictions are also placed in the gener of the given finite order. The algorithm numerical results for orders one through	ing semigroups eveloped to gen stinct up to ating procedur was placed on four obtained	s of finite nerate, for isomorphism re to produ n the compu-	e order is c any specified n. Additional ice all groups iter and the	
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